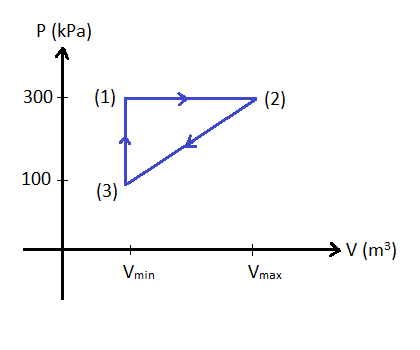
**Homework 2 Solutions**

**Problem 1.** Consider a steam engine, like the one we discussed in class, consisting of n = 3mol of water vapor. Let it operate between minimum/maximum temperatures T = 0°C, and T = 800°C, provided by cold and hot reservoir. And let it operate between minimum/maximum pressures p = 100kPa and p = 300kPa.



(a) Without doing any calculations, can you fill in the following table?

|  |  |  |  |
| --- | --- | --- | --- |
| **Process** | **1→2** | **2→3** | **3→1** |
| **Sign of W** | (+) | (-) | (0) |
| **Sign of Q** | (+) | (-) | (+) |

(a) What is Vmin?

The lowest temperature is the one that’s closest to the origin (point 3). So we have:



(b) What is Vmax?

The highest temperature is the one furthest from the origin (point 2). So,



(c) What is the temperature at point 1?

We can use same equation…



(d) What is the work done, and heat absorbed during process (1) → (2)?

Work is area under that line, so,



Heat can be calculated from 1st law:



(e) What is the work done and heat absorbed between process 2 → 3?

Work can be calculated as before, this time we’ll use the formula for the area under a trapezoid,



And to get heat, as usual 1st law:



(f) And what is the work done/heat absorbed between process 3 → 1?

This time, work is 0. And heat, comes from not the 2nd…not the 3rd…but the first law:



(g) What is total work done, heat absorbed, and heat exhausted?



(h) What should be the change in energy of the engine during this complete cycle? And does this accord (sans rounding errors) with your calculations vis a vis Wtotal, Qh, Qc? This is a good check that you did the calculations correctly.

Total energy change of the engine should be zero since it returns to its same initial temperature. And our calculations accord with this since: Q – W = ΔE → (66.2 – 64.2) – 2.1 = 0 → -0.1 = 0, which is approximately true, neglecting tiny rounding error.

(i) What must be the change in entropy of the engine? And the reservoirs? Is the total positive (or zero at best) as the second law requires?

ΔSengine should be 0 since it returns to its initial state.

ΔShot reservoir is: ΔSh = -Qh/Th = -66.2kJ/1073K = -62J/K,

ΔScold reservoir is: ΔSc = Qc/Tc = 64.2kJ/273K = 235J/K

Total ΔS = 0J/K – 62J/K + 235J/K = 173J/K, which is positive.

(j) What is the efficiency of our engine?

This is:



(k) How much heat would have to be input to raise a 500kg weight up through a vertical distance of 40m?



**Problem 2**. That’s not so hot. Playing around with P-V cycle shapes, operating between the same two reservoirs, we construct an engine with 40% the maximum efficiency.

(a) What is its efficiency?

Max efficiency is given by



And so our new engine has an efficiency of:



(b) Our engine is being used instead to rotate a turbine, to generate electricity (via mechanism to be discussed perhaps in PHY 123). We need it to generate work at the rate of 5MW. What rate of heat input is required to power our engine? What is the rate of heat exhaustion to the environment (should use 1st law to connect work, heat input, and heat output)?

So then,



And from the 1st law:

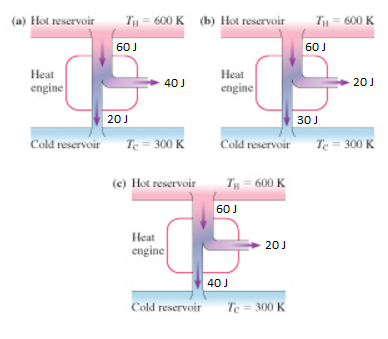


(c) The exhaust heat must be dumped somewhere. A nearby pond is a good choice! Suppose the pond has surface area A = 10 000m2, and depth h = 10m. The density of water is ρ = 1000 kg/m3, and its heat capacity is c = 4.18kJ/kg∙C. How many hours until the pond’s temperature increases by 1°C?

The mass of water in the pond is m = ρV = 108kg. And so then, applying the first law to the pond,



**Problem 3.** State which, if any, of the first and second laws of thermodynamics, the following engine cycles violate. Please justify ☺.



1. Since 60 = 40 + 20, the first law is satisfied, and since -60/600 + 20/300 = -0.033 < 0, the second law is violated.
2. Since 60 ≠ 20 + 30, the first law is violated. But since -60/600 +30/300 = 0 ≥ 0, the second law is barely satisfied.
3. Since 60 = 20 + 40, the first law I satisfied. And since -60/600 + 40/300 = 0.033 ≥ 0 the second law is satisfied.

**Problem 4.** You go out for a 6 mile jog. Ignoring internal work…

(a) How much work do you do?

Work is:



(a) Approximately how many kcalories do you burn?



(b) How much heat do you give off (assuming your temperature remains constant).



**Problem 5**. Suppose you use a stair climber, and climb the equivalent of 850m. And further that your mass is about 70kg, your efficiency about 20%, and your heat capacity about 3.5 kJ/kg∙K. Ignore internal work…

(a) How many kcalories do you burn?

Calories burned is just



(b) If you wore a thick jacket to eliminate evaporative cooling and to minimize thermal radiation and conduction as means of cooling off how much would your core temperature have increased?

If you eliminate all heat transfer then we’ll have:



(c) If your core temperature increases about 3 degrees, and you lose heat through evaporation of sweat, how much water would you lose (in kg).

And then if we assume heat transfer via evaporation,



**Problem 6.** Suppose you’re outside in the cold (T = -20C). How thick a jacket will you need to stay warm (meaning your temperature doesn’t change) if its material has thermal conductivity k = 0.04 W/m∙K? You can take your temperature to be 35C. And you can suppose that the area of the jacket is A = 0.7m2. Just consider heat transfer via conduction.



**Problem 7**. You can be modelled as a cylinder of height h = 1.7m, and radius R = 12cm, with skin temperature T = 35C. At what ambient temperature would you feel most comfortable? In other words, at what ambient temperature (in Farenheit) would you radiate heat at just the right pace to keep your temperature constant? Assume radiation is the only source of heat loss and takes place via all surfaces, and that ε = 1.



**Problem 8.** A Carnot refrigerator operates between reservoirs at 50C and -15C. If it uses a power of 15 Watts, how long will it take to cool 400mL of soda from 20C to 5C? You make take the soda to have the thermodynamic properties of water for these purposes (and note 1mL = 1g)

First we need to figure out the rate at which the refrigerator extracts heat from its interior. We have:



and also,



and now applying the first law to the soda,

